Eur. Phys. J. B **50**, 133–136 (2006)
DOI: 10.1140/epjb/e2006-00134-9

THE EUROPEAN
PHYSICAL JOURNAL B

Persistence in financial markets

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Received 24 October 2005 / Received in final form 7 December 2005
Published online 12 April 2006 –

Abstract. Persistence is studied in a financial context by mapping the time evolution of the values of the shares quoted on the London Financial Times Stock Exchange 100 index (FTSE 100) onto Ising spins. By following the time dependence of the spins, we find evidence for power law decay of the proportion of shares that remain either above or below their 'starting' values. As a result, we estimate a persistence exponent for the underlying financial market to be $\theta_f \sim 0.5$.

PACS. 05.20.-y Classical statistical mechanics – 05.50.+q Lattice theory and statistics (Ising, Potts, etc.) – 75.10.Hk Classical spin models – 75.40.Mg Numerical simulation studies

1 Introduction

In its most general form, persistence is concerned with the fraction of space which persists in its initial state up to some later time. The problem has been extensively studied in recent years for pure spin systems at both zero [1–4] and non-zero [5] temperatures.

For example, in the non-equilibrium dynamics of spin systems at zero-temperature, the system is prepared in a random state at $t = 0$ and the fraction of spins, $P(t)$, that persist in the same state as at $t = 0$ up to some later time t is studied. For the pure ferromagnetic two-dimensional Ising model the persistence probability has been found to decay algebraically [1–4]

$$P(t) \sim t^{-\theta}, \tag{1}$$

where $\theta \sim 0.22$ is the non-trivial persistence exponent [1–3].

The value of θ depends on both the spin [6] and spatial [3] dimensionalities; see Ray [7] for a recent review.

At non-zero temperatures [5], consideration of the global order parameter leads to a value of $\theta_{global} \sim 0.5$ for the two-dimensional Ising model.

Very recently, disordered systems [8–10] have also been studied and have been found to exhibit different persistence behaviour to that of pure systems.

Persistence has also been studied in a wide range of experimental systems and the value of θ ranges from 0.19 to 1.02 [11–13]. Much of the recent theoretical effort has gone into obtaining the numerical value of θ for different models.

In this work we present the first estimate for a persistence exponent extracted from *financial* data.

Long-range correlations in persistent and anti-persistent random walks were first discussed by Mandelbrot [14]. Zhang [15] has presented empirical evidence to support that daily returns in composite indices are not completely randomized. Here, on the other hand, we study the behaviour of the constituent stock prices.

2 Financial markets

A financial market is an example of a complex many-body system exhibiting many of the characteristics found in model systems studied in statistical physics.

There is an element of both co-operation and 'frustration' [16] in the movement of share values. For example, share values of companies in the same sector tend to move in the same direction (either up or down) when subjected to identical external events. A typical case here would be the movement in the value of shares in oil companies on the news of over/under production. On the other hand, there are also companies whose share values move in opposite directions given the same event. Here, typical examples would be the reactions in the share values of companies from the retail and banking sectors on the news of an increase/decrease in interest rates.

In this work we make no assumptions about any underlying model systems. Rather, we treat the historical share values of the companies over time as the outcomes of some 'experiment'.

The financial market we study is the set of companies quoted on the London Financial Times Stock Exchange 100 (FTSE 100) share index.

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Table 1. A mapping onto Ising spins of the end of day closing share prices for a typical company quoted on the FTSE 100.

Date	t	Closing price	$S_i(t)$
24 February 1995	0	166.00	Base price
27 February 1995	1	166.50	+1
28 February 1995	2	172.75	+1
1 March 1995	3	167.00	+1
2 March 1995	4	161.75	−1

3 Method

The data used in this study was obtained from Datastream [17], a database of financial information, and refers to the end of day prices over the randomly chosen ten year period from 24 February 1995 to 1 February 2005. The data were mapped onto Ising spins using the procedure outlined below.

The 'base' share price, $(P_i^b(t=0), i = 1\ldots, 100)$, of each of the companies appearing in FTSE 100 at the end of trading on 24 February 1995 was noted to 2 decimal places. All prices for the shares used in this work were taken to be the closing values at the end of trading. At $t = 1$ (the end of trading on the next day) the share price of each company, $P_i(t=1), i = 1\ldots, 100$, was compared with the corresponding base price.

We allocate a value $S_i(t=0) = +1$ if $P_i(t=1) \geq P_i^b(t=0)$ and a value of $S_i(t=0) = -1$ if $P_i(t=1) < P_i^b(t=0)$. Table 1 gives a typical example of the mapping. Note that the value of the spin is determined with reference to the base price and not the previous closing price. Furthermore, in this work we disregard all fluctuations which may have taken place during the day and simply use the end of closing prices. In the example discussed in Table 1, as the spin has 'flipped' when $t = 4$, the value of $S_i(t \geq 4) = -1$, *irrespective* of subsequent closing prices.

The values $\{S_i(t=0), i = 1, \ldots 100\}$ form the initial configuration for our 'spin' system. All of the subsequent 10 years worth of data was converted into possible values of Ising spins, $S_i(t)$, using the share values at $t = 0$ as the base. As a result, we are able to use $S_i(t)$ to track the value of the underlying share relative to its base price. It is worth noting that companies have to satisfy certain qualification criteria before they are included in the FTSE 100 [18]. As a result, in practice, a given company's presence in the FTSE 100 can fluctuate from year to year. In our analysis we restricted ourselves to the core set of companies remaining in the FTSE 100 throughout the time period under consideration.

Hence, the *first time* $S_i(t) \neq S_i(t=0)$ corresponds to the underlying share value either going above ($S_i(t) = +1$) or below ($S_i(t) = -1$) the base price also for the first time. This gives us a direct analogy with the persistence problem that has been extensively studied in spin systems.

At each time step, we count the number of spins that still persist in their initial ($t = 0$) state by evaluating [19]

$$n_i(t) = (S_i(t)S_i(0) + 1)/2. \tag{2}$$

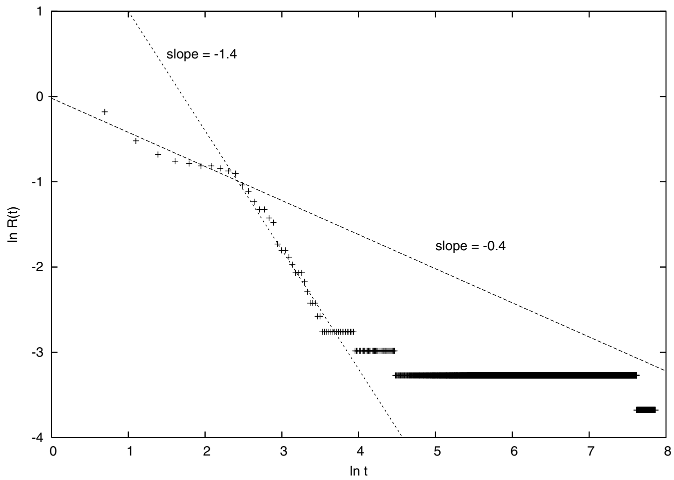

Fig. 1. A plot of $\ln R(t)$ against $\ln t$ for the data commencing 24 February 1995. The two straight lines are guides to the eye and have slopes −0.4 and −1.4 as indicated.

Initially, $n_i(0) = 1$ for all i. It changes to zero when a spin flips (that is, the underlying share price goes above/below the base price) for the first time. Note that once $n_i(t) = 0$, it remains so for all subsequent calculations.

The total number, $n(t)$, of spins which have never flipped until time t is then given by

$$n(t) = \sum_i n_i(t). \tag{3}$$

A key quantity of interest is $R(t)$, the density of non-flipping spins [1]

$$R(t) = n(t)/N, \tag{4}$$

where N is the number of companies monitored (note that N is not necessarily 100 for the reasons outlined earlier). The actual values of N used are stated below. However, it's worth noting that, in principle, we are dealing with a model system where the spins are interacting with all other spins. As a result, we do not believe that our system of spins is too small.

4 Results

We now discuss our results. In this initial study, three different time periods were considered:

a) 24 February 1995 to 1 February 2005 ($N = 79$);
b) 9 January 1996 to 28 December 2000 ($N = 79$);
c) 3 January 2000 to 3 January 2005 ($N = 92$).

These sets were selected at random. For each time period, the initial and subsequent spin configurations were generated as outlined above and the resulting data analysed for persistence behaviour.

In Figure 1 we show a log-log plot of the density of non-flipping spins against time for the first set of data commencing 24 February 1995. There is evidence for an initial power-law decay (slope = −0.4), leading to a faster subsequent decay with slope = −1.4.

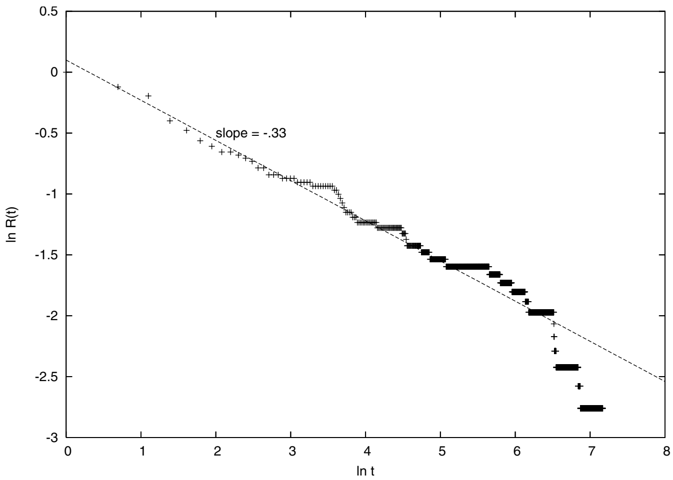

Fig. 2. A log-log plot of the data commencing 9 January 1996. The straight line (guide to the eye) has a slope of −0.33.

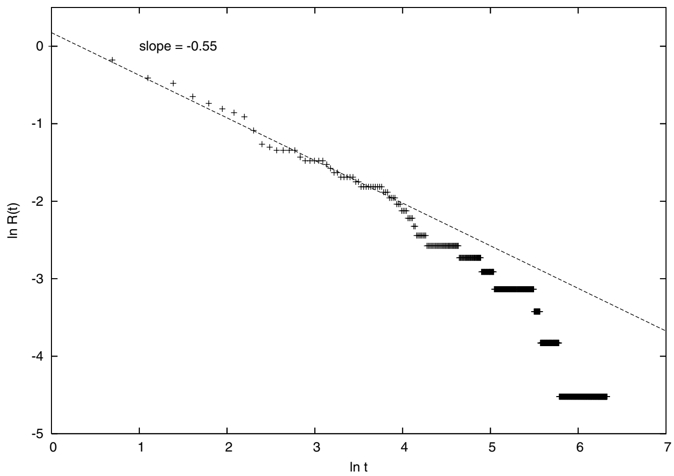

Fig. 3. A log-log plot of the data commencing 3 January 2000. Once again, the straight line (slope −0.55) is a guide to the eye.

A key feature of Figure 1 is that nearly all of the spins have flipped after $\ln t \approx 4$. This corresponds to approximately 30−50 days of trading on the markets.

Note that we are not distinguishing between those shares that remain above or below their base values. There are also a few shares (in this case just 2) that still persist in their initial state over the entire observation period of 10 years. To investigate the problem further, the same data was partitioned into essentially two 5-year blocks as outlined above. The analysis was repeated on each of the two sets of data. In Figure 2 we plot $\ln R(t)$ against $\ln t$ for the data commencing 9 January 1996. Note that for this plot the base prices are determined by close of trading on 9 January 1996. Once again, there is clear evidence for a power-law decay. This time, however, the slope of the linear fit is −0.33.

Finally Figure 3 show a similar plot for the data commencing 3 January 2000. once again, we have evidence for initial power-law decay (slope = −0.55) and most of the spins appear to have flipped after $\ln t \approx 4$.

Although, as expected, there is noise in the data, we see that in all three cases we have clear evidence of an initial power-law decay. It's also clear from the plots that nearly all of the spins have flipped after a fairly short period of time, corresponding to approximately 30−50 days of trading on the markets. However, there are a handful of spins which do not flip over the entire time period under consideration.

Furthermore, there appears to be a cross-over to a faster power-law decay for longer times. This cross-over is only really evident in Figure 1 as there is too much noise in the other 2 sets of data. The initial decay in Figure 1 has a slope of −0.4. The faster decay is indicated by the straight line which has a slope of −1.4. The initial power law decays are indicated by the straight lines in Figures 2 and 3. The cross-over could be a signature of the market reacting to external events such as significant interest rate variations or political news.

From the linear fits, we can extract a value of θ_f ranging from 0.33 to 0.55. As a consequence, we estimate the persistence exponent for the financial data to be ∼0.5. We believe this to be the first estimate of a persistence exponent from financial data.

Our value for θ_f is not inconsistent with the value obtained from computer simulations of the $2D$ Ising model at a non-zero temperature [5]. This is an intriguing result as we have made no assumptions whatsoever about the underlying model which gives rise to the financial data. Of course, in our analysis, the value of each $S_i(t)$ incorporates the overall performance of the shares of the underlying company relative to the base value.

5 Conclusion

To conclude, we have used a novel mapping to map the share values quoted on the London Financial Times Stock Exchange 100 share index onto Ising spins. As a result, we extracted a value of ∼0.5 for the persistence exponent. This should be regarded as an initial estimate and further work is required to confirm the value. It should be noted that, out of necessity, we worked with end of day closing prices. Ideally, it would be better to use tick-data. It's remarkable that our value is not inconsistent with the value of the persistence exponent obtained for the $2D$-Ising model at non-zero temperature. This observation justifies further investigation.

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